FUZZY ADAPTIVE EXTENDED KALMAN FILTER

Vasko Sazdovski, Tatjana Kolemishevska-Gugulovska, Mile Stankovski

Institute of ASE at Faculty of EE,, St. Cyril and Methodius University, MK-1000, Skopje, Republic of Macedonia; E-mail: sazdo79@yahoo.com,[milestk, tanjakg]@etf.ukim.edu.mk

Abstract: Kalman filtering is a method for estimating state variables of a dynamic systems recursively from noise-contaminated measurements. For systems with nonlinear dynamics, a natural extension of the Linear Kalman Filter (LKF), called Extended Kalman filter (EKF) is used. The Kalman filter represents one of the most popular estimation techniques for integrating signals from navigation systems, like Inertial Navigation System (INS) and Global Positioning System (GPS). However, a significant difficulty in designing a Kalman Filter (refers to both LKF and EKF) can often be traced to incomplete *a priori* information about R and Q matrices. It has been shown that incorrect *a priori* information can lead to practical divergence of the filter. The use of fuzzy-rule based adaptation scheme to cope with divergence problem is explored. The Fuzzy Logic Adaptive Controller (FLAC) was implemented in Integrated INS/GPS Navigation Systems to detect the uncertainties, adapt the Kalman Filter on-line and prevent divergence.

Keywords: Fuzzy logic, Kalman filters, Extended Kalman filters, inertial navigation, Global positioning systems, simulation.

1. INTRODUCTION

Kalman filtering is a method for estimating state variables of a dynamic systems recursively from noise-contaminated measurements. The filter determines the system's present state by optimally combining theoretical estimates with measurements noise characteristics based on the knowledge of the system model. This is the well known Linear Kalman filter (LKF) for state estimation of linear systems. For systems with nonlinear dynamics, a natural extension of the LKF called Extended Kalman filter (EKF) is used.

The Kalman filter represents one of the most popular estimation techniques for integrating signals from short-term high performance navigation systems (like Inertial Navigation System, INS) with reference systems exhibiting long-term stability (like Global Positioning System, GPS). For the integration of GPS and INS, because the systems are nonlinear in nature we use the EKF where the GPS errors are represented by the measurement noise covariance matrix R, and the INS errors are represented by process covariance matrix Q. However, a significant difficulty in designing a Kalman Filter (refers to both LKF and EKF) can often be traced to incomplete *a priori* information about R and Q. In most practical applications these matrices are initially estimated or even unknown. Incorrect *a priori* information can lead to practical divergence of the filter, resulting in a difference in the theoretical and actual behaviour of the filter.

The use of fuzzy-rule based adaptation scheme to cope with divergence problem caused by the insufficiently known *a priori* filter statistics is explored. Integrated INS/GPS Navigation System with implemented FLAC is used for navigation, guidance and control of Unmanned Aerial or Ground Vehicles. The navigation of mobile robots requires fast, accurate, on-line control algorithms with reliable navigation parameters.

2. NAVIGATION SYSTEMS

2.1. Inertial Navigation System (INS)

The INS algorithm integrates the accelerations and angular rates provided by an Inertial Measurement Unit (IMU) to compute the position, velocity, and attitude (PVA) of the vehicle (Lin 1991).
The algorithm takes into account the geoid shape and a gravity model. There are many problems with noise and unbounded error that must be handled to get any meaningful result out of the INS. The information provided by the IMU such as body accelerations are transformed to navigation frame and gravity vector is subtracted. The resulting acceleration vector is integrated with respect to time and we get the velocity of the vehicle. The velocity vector is then integrated and we can read the position of the vehicle.

2.2 Global Positioning System (GPS)

Global Positioning System (GPS) can be regarded as a new navigation sensor. GPS provides range and range-rate measurements. The primary role of GPS is to provide highly accurate position and velocity worldwide, based on range and range-rate measurements. The acceleration vector is then determined from positions at different time epochs, by differentiation of these positions with respect to time. Position accuracy of GPS pseudo-range absolute positioning is affected by measurement noise (few metres) and signal errors like: multipath of the signal, ionosphere delays, troposphere delays, signal attenuation, ephemeris error, satellite clock error and receiver clock error (Ronback, 2002). Also, the GPS signal is susceptible to jamming. For many vehicle navigation systems, GPS is insufficient as a stand alone position system.

3. KALMAN FILTER

The Kalman filter (Gene et al. 2000) assumes that the random process which has to be estimated is of the form:

$$\begin{aligned} \dot{x} &= Fx + Bu + Gw \\ z &= Hx + Du + v \end{aligned} \tag{1}$$

where, x is a state value, u is a control effort, w is white noise with known covariance, z is a noisy measurement sample, D is the direct transmission of the input to the output, H is the ideal (noiseless) connection between the measurement and the state, and v is measurement error. This process can be modeled discretely in the following form, assuming there are not control inputs u to the system.

$$\begin{aligned} x_{K+1} &= \Phi_K x_K + w_K \\ z_K &= H_K x_K + v_K \end{aligned} \tag{3}$$

The system error is defined as:

$$e_K^- = x_K - \hat{x}_K^-$$

where, $\hat{x}_k^-$ is the best estimate prior to receiving a measurement at time t_K.

The error covariance matrix at this time is:

$$P_K^- = E[e_K^- e_K^{-T}] = E[(x_K - \hat{x}_K^-)(x_K - \hat{x}_K^-)^T] \tag{4}$$

where E is the expectation. Now a linear blending of both the estimate and the measured values (residuals) is taken.

$$\hat{x}_K = \hat{x}_K^- + K_K(z_K - H_K \hat{x}_K^-) \tag{5}$$

where $\hat{x}_K$ is the new updated estimate, z is measured values (residuals), and K is a weighted value that determines the amount of error between the measured value and the best estimate. This gain is referred to as the *Kalman gain* which is capable of changing value over time. Now looking at the error covariance of this new updated estimate, we get the following equation:

$$P_K = E[e_K e_K^T] = E[(x_K - \hat{x}_K)(x_K - \hat{x}_K)^T] \tag{6}$$

Now, after some algebra the following expression results for the error covariance matrix:

$$P_K = (I - K_K H_K) P_K^- (I - K_K H_K)^T + K_K R_K K_K \tag{7}$$

This is a general expression for updating the error covariance matrix, and it applies for any value of K. The resulting gain K is computed by the equation:

$$K_K = P_K^- H_K^- (H_K P_K^- - H_K^T + R_K)^{-1} = \frac{P_K^- H_K^-}{H_K P_K^- - H_K^T + R_K}. \tag{8}$$

Looking at equation (8), we see that as the measurement error covariance approaches zero, the gain K weights the residual more heavily. Specifically,

$$\lim_{R_K \to 0} K_K = H^{-1}.$$

On the other hand, as the *a priori* estimate error covariance approaches zero, the gain K weights the residual less heavily. Specifically,

$$\lim_{P_K^- \to 0} K_K = 0.$$

4. EXTENDED KALMAN FILTER

As described above, the Kalman filter addresses the general problem of trying to estimate the state of a discrete-time controlled process that is governed by a *linear* stochastic difference equation. We know that real processes are highly nonlinear in nature, in other words we cannot find a linear system. The question know is, what happens if the process to be estimated and (or) the measurement relationship to the process is non-linear? Some of the most interesting and successful applications of Kalman filtering have been such situations.
Natural extension of the Linear Kalman filter or Kalman filter that linearizes about the current mean and covariance is referred to as an *Extended Kalman filter* or EKF. Extended Kalman filter is based on Taylor series, where in the face of non-linear relationships we linearize the estimation around the current estimate using the partial derivatives of the process and measurement functions to compute estimates.

5. FUZZY LOGIC CONTROL

Fuzzy logic is a fascinating area of research because it does a good job of trading off between significance and precision, something that humans have been managing for a very long time. Fuzzy logic control is a control method based on fuzzy logic and today is very effective and widely used control concept of complex, nonlinear dynamic systems.
Just as fuzzy logic can be described as ”computing with words rather than numbers„ ; fuzzy logic control can be described simply by ”control with sentences rather than equations„.
The basic configuration of the fuzzy logic controller is shown in Fig. I.

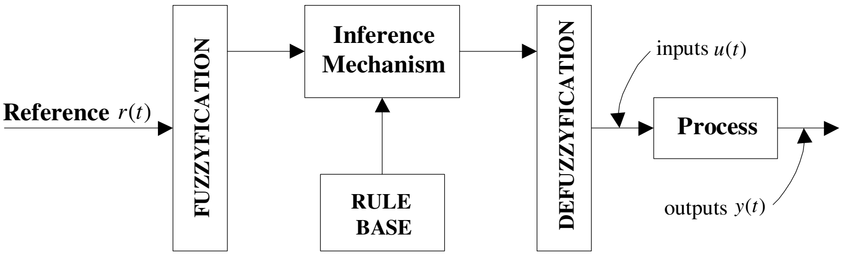


Fig. I. Fuzzy logic Controller Architecture

5.1 Rule Base

Specifically, the fuzzy rule-base comprises the following fuzzy If-Then rules:
IF x_1 is A_1^l and … and x_n is A_n^l, THEN y is B^l,
where A_i^l and B^l are fuzzy sets in $U_i \subset R$ and $V \subset R$, respectively, and $x = (x_1, x_2, \ldots, x_n)^T \in U$ and $y \in V$ are the input and output (linguistic) variables of the fuzzy system, respectively.

5.2 Inference Mechanism

The premises of all the rules are compared to the controller inputs to determine which rules apply to the current situation. The ”matching„ process involves determining the certainty that each rule applies.

5.3 Fuzzification

The fuzzification process is the act of obtaining a value of an input variable and finding the numeric values of the membership function(s) that are defined for that variable.

5.4 Defuzzification

Defuzzification operates on the implied fuzzy sets produced by the inference mechanism and combines their effects to provide the ”most certain„ controller output.
Perhaps the most popular method is the center of Gravity (COG) method also known as centroid calculation, which returns the center of area under the curve. There are other methods like: bisector, middle of maximum (the average of the maximum value of the output set), largest of maximum, and smallest of maximum.
Center of Gravity method:

$$u^{crisp} = \frac{\sum_i b_i \int \mu_{(i)}}{\sum_i \int \mu_{(i)}}, \tag{9}$$

where b_i, is the center of the membership function of the consequent of rule (i), $\int \mu_{(i)}$ is area under membership function $\mu_{(i)}$.

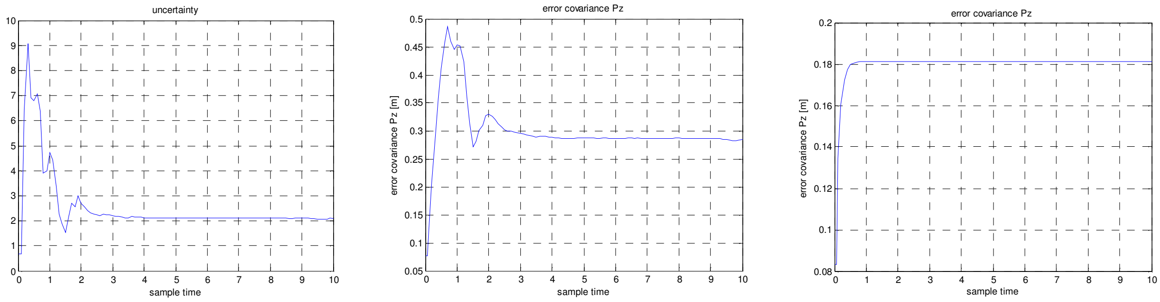
6. FUZZY LOGIC ADAPTIVE CONTROLLER

A lot of a priori information's such as $\hat{x}_0, P_0, R, Q$ are essential for designing the Kalman filter. In most cases, these matrices are initially estimated or even unknown. The problem here is that the optimality of the estimation algorithm is closely connected to the quality of *a priori* information's. It has been shown that insufficiently known *a priori* filter statistics can reduce the precision of the estimated filter states. In addition, incorrect *a priori* information can lead to practical divergence of the filter, resulting in a difference in the theoretical and actual behaviour of the filter. There are two kinds of divergence: apparent divergence and true divergence (Sasiadek, 2002). In the apparent divergence, the actual estimate error covariance remains bounded, but it approaches a larger bound than does predicted error covariance. In true divergence, the actual estimation covariance eventually becomes infinite. The divergence due to modeling errors is critical in Kalman filter application. If, the Kalman filter is fed information that the process behaved one way, whereas, in fact, it behaves another way, the filter will try to continually fit a wrong process. When the measurement situation does not provide enough information to estimate all the state variables of the system, in other words, the computed estimation error matrix becomes unrealistically small, and the filter disregards the measurement, then the problem is particularly severe. Thus, in order to solve the divergence due to modeling errors, we can estimate unmodeled states, but it adds complexity to the filter and one can never be sure that all of the suspected unstable states are indeed model states (Sasiadek, 2002). Another possibility is to add process noise. It makes sure that the Kalman filter is driven by white noise, and prevents the filter from disregarding new measurement. If the Kalman filter is based on a complete and perfectly tuned model, the residuals or innovations should be a zero-mean white noise process. If the residuals are not white noise, there is something wrong with the design and the filter is not performing optimally. We assume that uncertainties α, or time varying parameters exist in matrix R.

The Fuzzy Logic Adaptive Controller (FLAC) is used to detect these uncertainties, adapt the EKF with that the whole Integrated INS/GPS Navigation System on-line and prevent divergence. In the FLAC variance and the mean of residuals are used as inputs for the fuzzy inference engine while the uncertainties in measurement noise covariance matrix are used as output. Variance is a very useful statistical property for random signals because if we knew the variance of a signal that was otherwise supposed to be "constant" around some value the mean, the magnitude of the variance would give us a sense how much noise or uncertainty is in the signal. Generally, when we have great uncertainties the variance is becoming large, and mean value is moving away from zero, the EKF is becoming unstable and divergence problems occurs. By detecting an appropriate value of the uncertainties FLAC continually adjusts the noise strengths in the filter's internal model and adapt the EKF optimally trying to keep the innovation sequence acting as zero-mean white noise.

7. SIMULATION EXPERIMENTS AND RESULTS

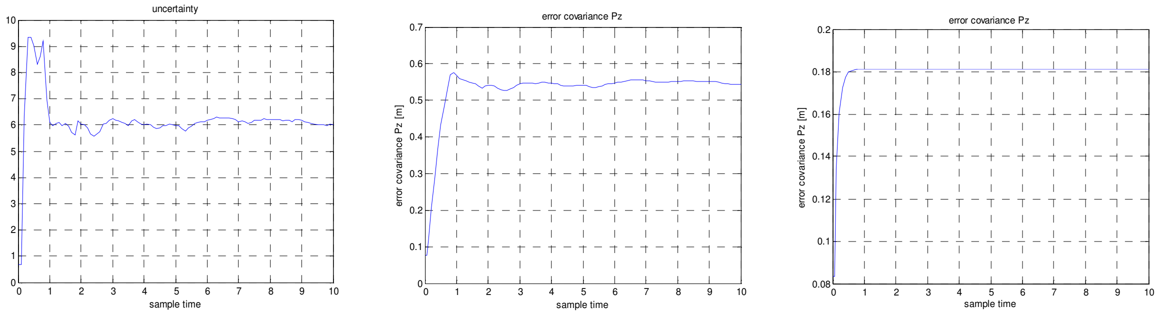
In order to see the FLAC in act we compare the behaviour of the error covariance P_Z in two cases. In the first case matrix R is constant and we don't use the adoption scheme. In the second case, we use the FLAC to detect the uncertainties α, and to adapt the error covariance P_Z. The designed standard deviation of GPS measurement R is 5 meters. The designed standard deviation of Q for INS is 0.0012 meters. When we carried out the simulation experiments we change the value of the measurement noise covariance. So we have "true" measurement noise covariance R_T, that is not equal to the designed covariance in the EKF. And we have $R_T = 3R, 4R, 6R$ in other words the "true" measurement noise covariance is three, four, six times greater then the designed covariance.



a) b) c)

Fig II. Detection of the uncertainty when $R_T = 2R$

Figure II shows the FLAC in act. From the case a) we can see the detected uncertainty $\alpha = 2$ and the difference between the adapted, case b) and not adapted error covariance P_Z, case c).



a) b) c)

Fig III. Detection of the uncertainty when $R_T = 6R$

Figure III shows the FLAC in act. From the case a) we can see the detected uncertainty $\alpha = 6$ and the difference between the adapted, case b) and not adapted error covariance P_Z, case c). From figure III we can also see that redesigned or adapted error covariance P_Z is three times greater than the designed or not adapted error covariance.

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